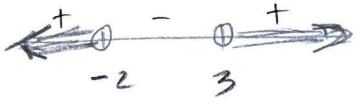
Sections 2.6 & 2.7 – I.C.E #2

Part A: For #1 – 4, be sure to use a sign chart to find the appropriate intervals for your solution set:

1) Solve $x^2 - x > 6$ and answer using interval notation

$x^2 - x - 6 > 0$

$(x-3)(x+2) > 0$



we want > 0

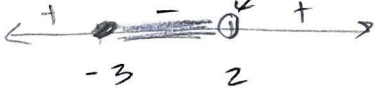
$(-\infty, -2) \cup (3, \infty)$

2) Solve $\frac{2(x+3)}{x-2} \leq 0$ and answer using interval notation

$D: \{x \mid x \neq 2\}$

critical values: $-3, 2$

not in Domain!



we want ≤ 0

$[-3, 2)$

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3) Solve $\frac{x^2+9x+18}{x^2-2x-8} \geq 0$ and answer using interval notation

$= \frac{(x+6)(x+3)}{(x-4)(x+2)} \geq 0$ $\quad D: \{x \mid x \neq -2, 4\}$

critical values: $-6, -3, -2, 4$

$(-\infty, -6] \cup [-3, -2) \cup (4, \infty)$

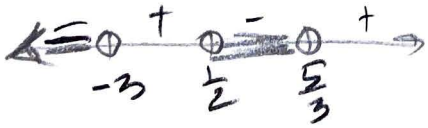
we want ≥ 0

4) Solve $\frac{4}{x+3} - \frac{2}{2x-1} < 0$ and answer using interval notation

$= \frac{4(2x-1)-2(x+3)}{(x+3)(2x-1)} = \frac{8x-4-2x-6}{(x+3)(2x-1)} = \frac{6x-10}{(x+3)(2x-1)}$

$= \frac{2(3x-5)}{(x+3)(2x-1)} < 0$ $\quad D: \{x \mid x \neq -3, \frac{1}{2}\}$

critical values: $-3, \frac{1}{2}, \frac{5}{3}$



$(-\infty, -3) \cup (\frac{1}{2}, \frac{5}{3})$

we want < 0

Sections 2.6 & 2.7 – I.C.E #2

Part B: Find all horizontal, slant, and vertical asymptotes for each function. Be sure to state your answers as equations of lines. Also find all x and y-intercepts and draw a sketch of the graph. Label where the asymptotes and the intercepts are located on your graph.

1) $f(x)=\frac{2x-3}{x-4}$

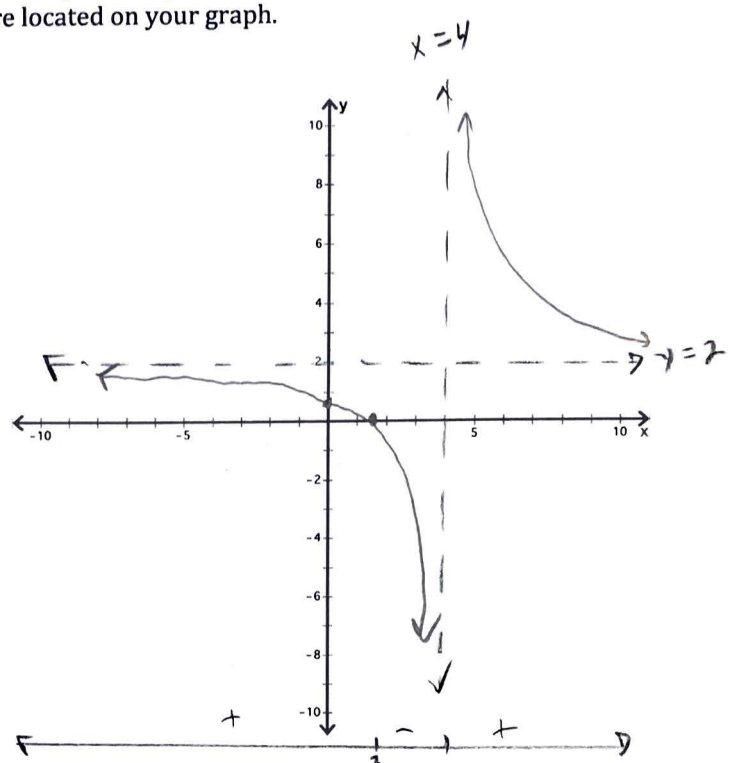
Hole? No!

VA: $x=4$

HA or SA: $y=2$

x-int: $(3/2, 0)$

y-int: $(0, 3/4)$



2) $f(x)=\frac{-x^3}{x^2-9}=\frac{-x^3}{(x+3)(x-3)}$

Hole? No!

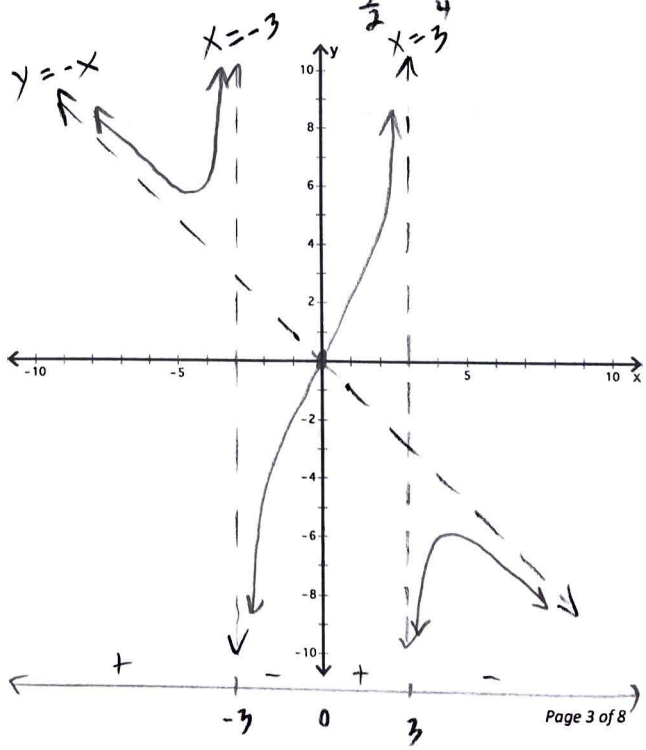
VA: $x=3, x=-3$

HA or SA: $y=-x$

x-int: $(0,0)$

y-int: $(0,0)$

$$x^2-9 \;\overline{)\; -x^3+0x^2+0x+0}$$
$$-(-x^3+0x^2+9x)$$
$$-9x+0$$



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3) $f(x)=\dfrac{2x^2+6x+4}{x^2-x-6} = \dfrac{2(x^2+3x+2)}{(x-3)(x+2)} = \dfrac{2(x+2)(x+1)}{(x+2)(x-3)}$

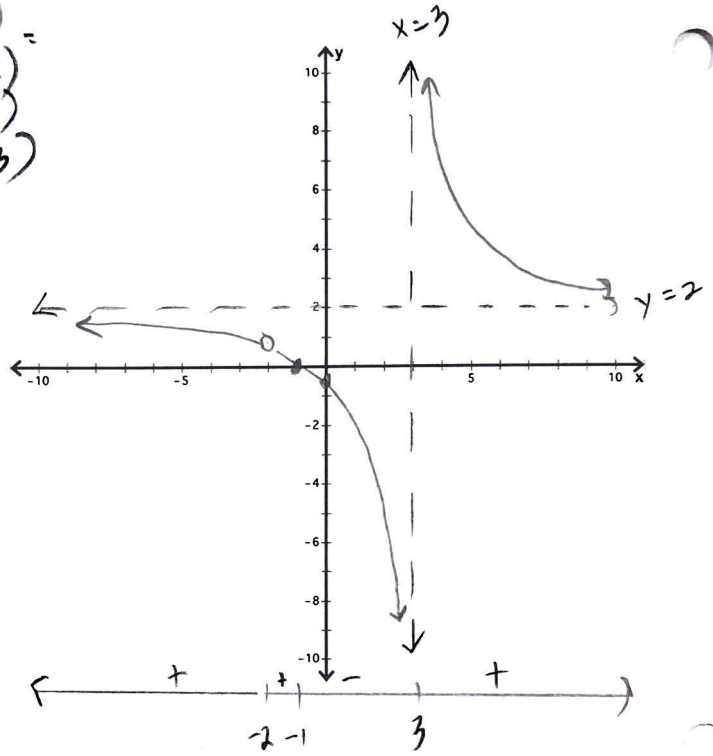
Hole? Yes! @ $x=-2$ $(-2, 2/5)$

VA: $x=3$

HA or SA: $y=2$

x-int: $(-1,0)$

y-int: $(0,-2/3)$



4) $f(x)=\dfrac{x^2-2x-8}{2x^2-10x+8} = \dfrac{(x-4)(x+2)}{2(x-1)(x-4)}$ $\frac{4}{4}$

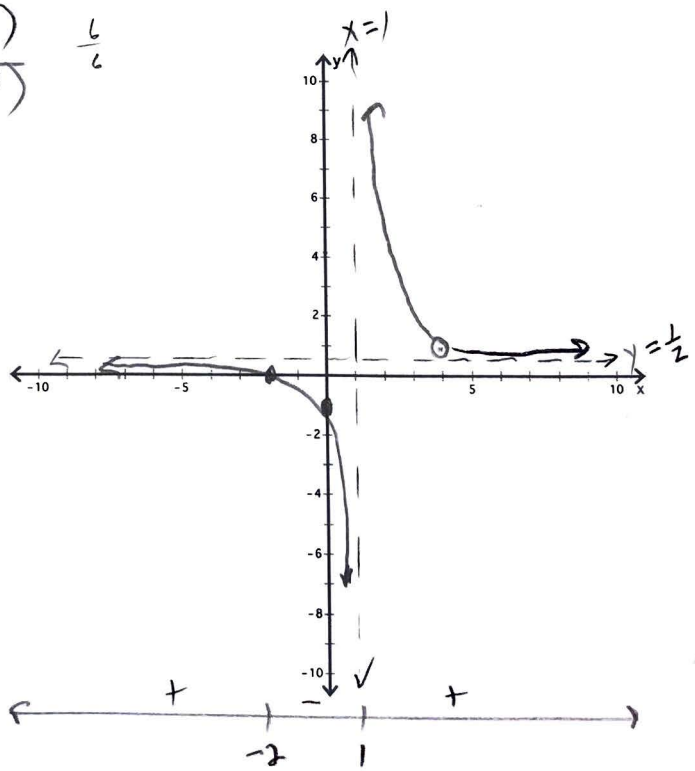
Hole? Yes! $x=4$ $(4,1)$

VA: $x=1$

HA or SA: $y=\frac{1}{2}$

x-int: $(-2,0)$

y-int: $(0,-1)$



Sections 2.6 & 2.7 – I.C.E #2

5) $g(x)=\frac{5}{x^2-16}=\frac{5}{(x+4)(x-4)}$

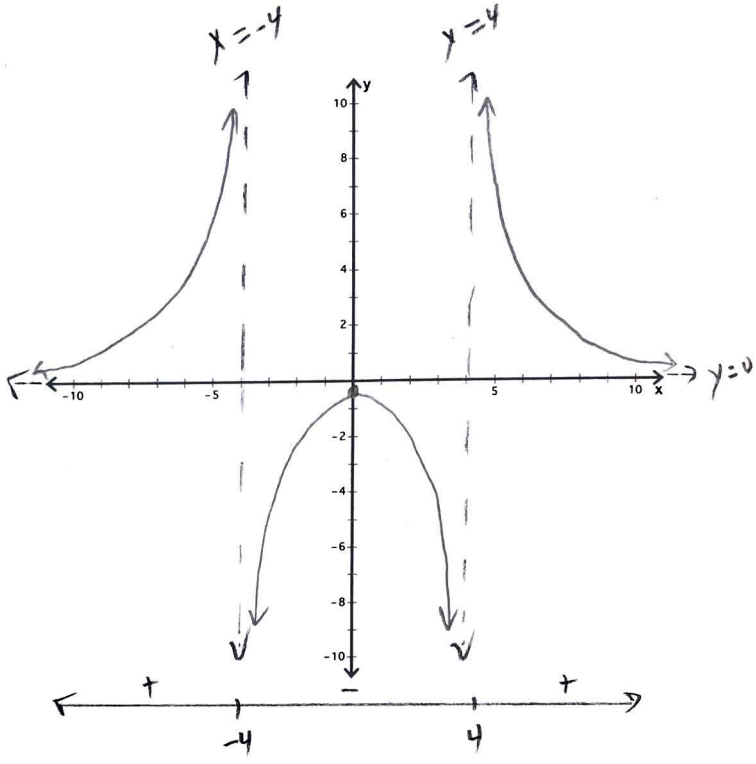
Hole? No

VA: $x=-4, x=4$

HA or SA: $y=0$

x-int: none

y-int: $(0,-5/16)$



6) $h(x)=\frac{-4x}{x^2-2}=\frac{-4x}{(x+\sqrt{2})(x-\sqrt{2})}$

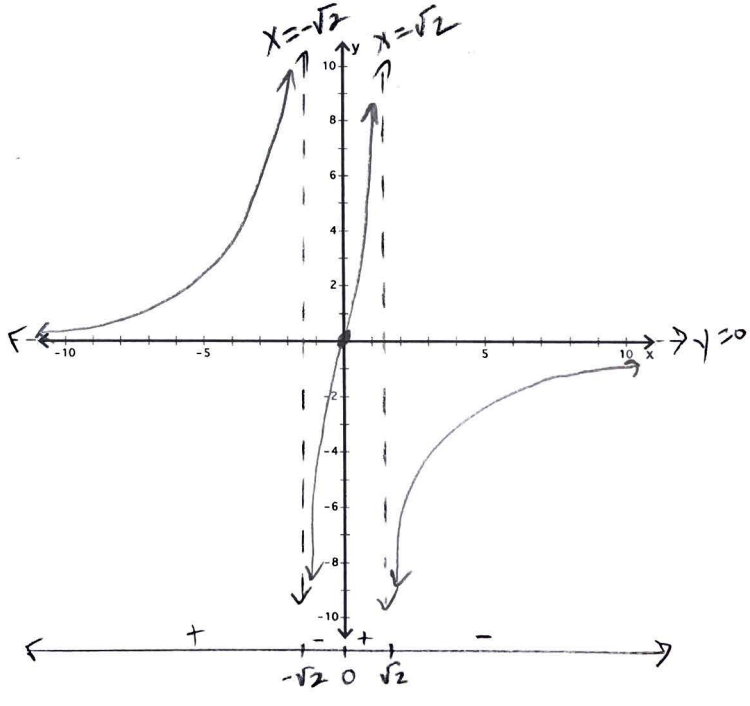
Hole? No

VA: $x=-\sqrt{2}, x=\sqrt{2}$

HA or SA: $y=0$

x-int: $(0,0)$

y-int: $(0,0)$



Sections 2.6 & 2.7 – I.C.E #2

7) $f(x)=\frac{3x^2}{x^2+5}$

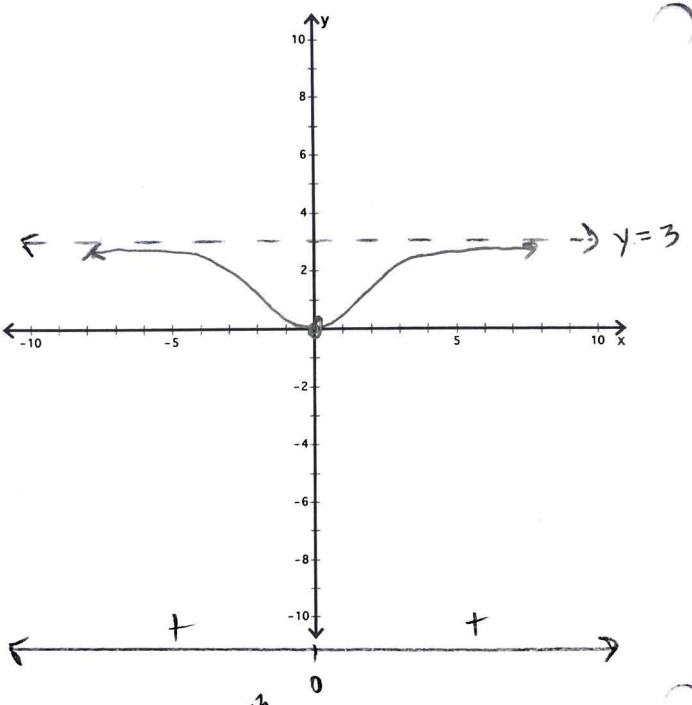
Hole? NO!

VA: none

HA or SA: $y=3$

x-int: (0,0)

y-int: (0,0)



8) $k(x)=\frac{-3}{x^3+3x^2}$ = $\frac{-3}{x^2(x+3)}$

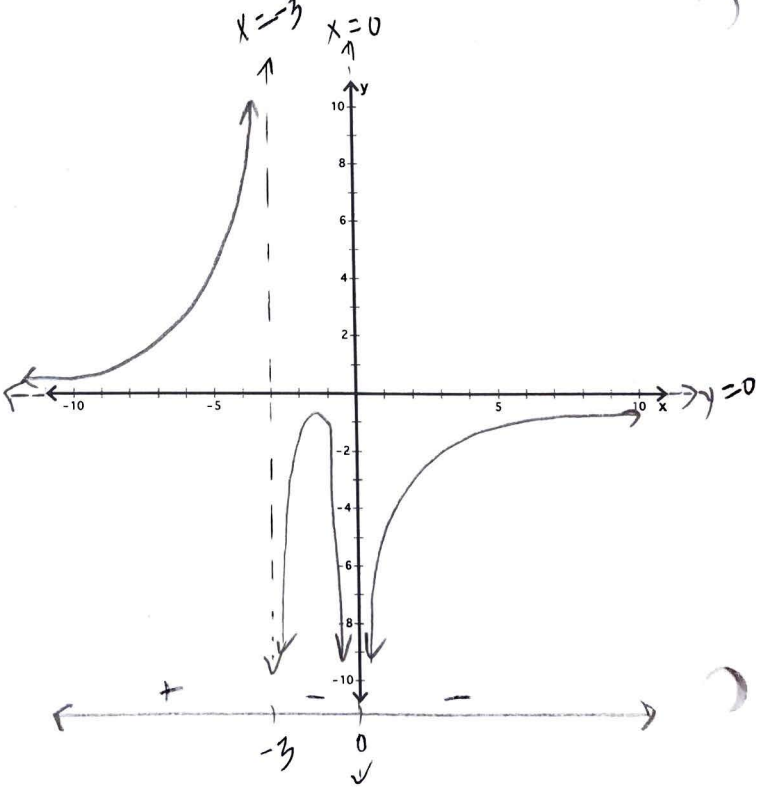
Hole? No!

VA: $x=0$, $x=-3$

HA or SA: $y=0$

x-int: none

y-int: none



Sections 2.6 & 2.7 – I.C.E #2

9) $f(x)=\dfrac{x^2-3x-12}{x+2}$ → not factorable → $\dfrac{-(-3)\pm\sqrt{(-3)^2-4(1)(-12)}}{2} = \dfrac{3\pm\sqrt{57}}{2}$

Hole? no!

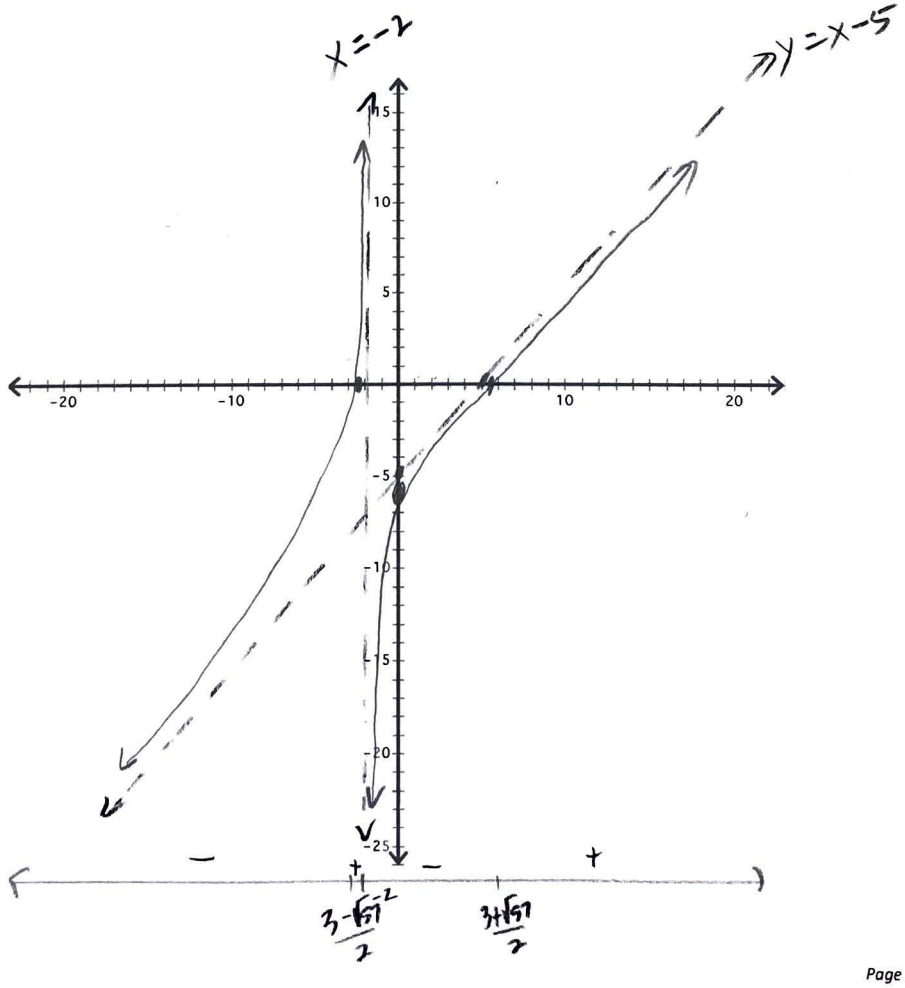
VA: $x=-2$

HA or SA: $y=x-5$

$-2 \mid 1 \quad -3 \quad -12$
$\quad\quad\quad -2 \quad 10$
$\quad 1 \quad -5 \quad -2$ ← ignore remainder

x-int: $\left(\dfrac{3+\sqrt{57}}{2},0\right),\left(\dfrac{3-\sqrt{57}}{2},0\right)$

y-int: $(0,-6)$



Sections 2.6 & 2.7 – I.C.E #2

10) $f(x)=\frac{3x^3-21x+18}{x^2-x-12}$ (hint: try x - 1 as a factor for the numerator)

Hole? Yes: $x=-3$ $\left(-3,\frac{60}{7}\right)$

VA: $x=4$

HA or SA: $y=3x+3$

x-int: $(1,0),(2,0)$

y-int: $(0,-3/2)$

$$f(x)=\frac{3(x-1)(x+3)(x-2)}{(x+3)(x-4)}$$

$+60$, -7

$$\begin{array}{r|rrrr} 1 & 3 & 0 & -21 & 18 \\ & & 3 & 3 & -18 \\ \hline & 3 & 3 & -18 & \boxed{0} \end{array}$$

$$\begin{array}{r} 3x+3 \\ x^2-x-12\overline{)\,3x^3+0x^2-21x+18} \\ \underline{-(3x^3-3x^2-36x)} \\ 3x^2+15x+18 \end{array}$$